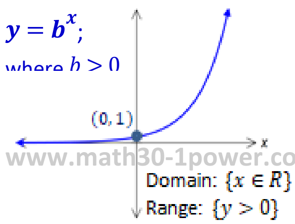
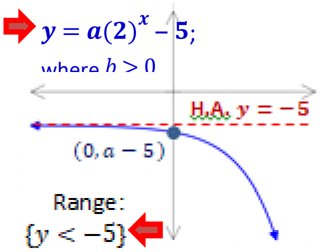
**1** First consider graph of $f(x)$, since the domain of the inverse is the range of $f(x)$. (They switch!) So let's re-phrase this question … what's the RANGE of $f(x)$?




*So the domain of the inverse is* $x < -5$ ANSWER: **C**

**2** $(b+1)^{\frac{1}{2}} = 3m$ First step – *convert to exp. form*

$\left((b+1)^{\frac{1}{2}}\right)^2 = (3m)^2$ To isolate $b$, square both sides (get that exp 1/'2 to be "1")

$\left((b+1)^{\frac{1}{2}}\right)^{2} = 9m^2$

$b + 1 = 9m^2$
$\quad -1 \quad -1$

$\mathbf{b = 9m^2 - 1}$ ANSWER: **A**

**3** $ax + b > 0$ $f(x) = log_b(ax + b)$

*Isolate* $x$ Whatever we're "logging" must be greater than 0. Can't log 0, can't log negatives!

$ax > -b$

$x > \frac{-b}{a}$ ANSWER: **A**

**NR#1** $f(0) = a(b^0) + d$ For $y$-intercept, set $x = 0$

$= a(1) + d$

$= \mathbf{a + d}$ ← CODE 4

$y = 0$ ← Asymptote of basic exp. graph $y = b^x$, shown above in answer to #1

$\mathbf{y = d}$ ← Horiz. Asymptote after being shifted vertically "d" units.

CODES 7 and 2 (either order) ANSWER: **472** or **427**

**4** $f(0) = log_b(a(\mathbf{0}) + b)$ For $y$-intercept, set $x = 0$

$= log_b(0 + b)$

$= log_b(b)$ ← $b^{\square} = b$?

$= \mathbf{1}$ ANSWER: **C**

**5** $\frac{3^{x^2+x}}{(3^{3})^{3x-1}} = 3(3^{-2})^{x-2}$ *Re-write all three terms in base 3*

$\frac{3^{x^2+x}}{3^{9x-3}} = 3^1 * 3^{-2x+4}$ *Simplify using exponent rules*

$3^{x^2+x-(9x-3)} = 3^{-2x+4}$ *Once both sides are fully simplified – set the exponents equal*

$x^2 + x - 9x + 3 = -2x + 4$

$\mathbf{x^2 - 6x - 2 = 0}$ ANSWER: **C**

← *c value!*

**6** $a^b = \frac{c}{2}$ ← First – isolate power term by dividing both sides by 2

$log_a\left(\frac{c}{2}\right) = b$ Then convert to log form to isolate "b"

$\mathbf{b = log_a\left(\frac{c}{2}\right)}$ ANSWER: **D**



**NR#2**

*Check* **Statement 1:**

$f(0) = a(2^{(0)-1}) + d$ ← *For y-int, set* $x = 0$

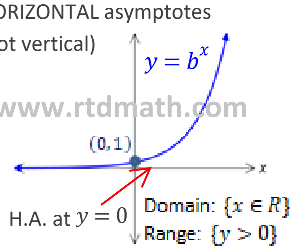
$= a(2^{-1}) + d$

$= a\left(\frac{1}{2}\right) + d$

$= \frac{\mathbf{a}}{\mathbf{2}} + \mathbf{d}$ **Statement 1 is TRUE**

*Check* **Statement 2:**

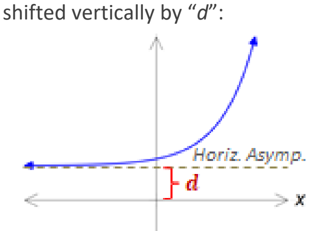
Exponential functions have HORIZONTAL asymptotes (not vertical)




**Statement 2 is FALSE**

*Check* **Statement 3:**

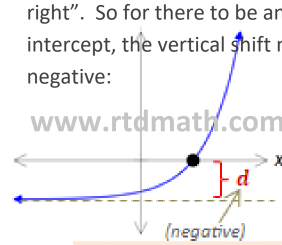
Consider the graph on the left shifted vertically by "*d*":




**Statement 3 is TRUE**

*Check* **Statement 4:**

Since $a > 0$ the graph must "rise right". So for there to be an $x$-intercept, the vertical shift must be negative:




**Statement 3 is FALSE**

*There can be an* $x$*-intercept*

*Check* **Statement 5:**

*See rationale for statement 4*

**Statement 5 is TRUE**

*Check* **Statement 6:**

*The domain of the inverse is the RANGE of* $y = f(x)$ ← *Which is NOT* $y \in R$ **Statement 6 is FALSE**

ANSWER: **135**

## 7

$y = ab^{\frac{t}{p}}$ ← *Use this formula, with $p = 1$ since we want a growth rate per 1 year*

$1239220 = 403\,319(b)^{45}$

end amount; Initial amount; # of years from 1971 to 2016

$\frac{1239220}{403\,319} = b^{45}$ *Isolate the power term*

$b^{45} \approx 3.072555$ *Take the "45th root" of both sides to solve for $b$*

$\sqrt[45]{b^{45}} \approx \sqrt[45]{3.072555}$

$b \approx 1.025$ *$b = 1 +$ growth rate*

*growth rate* — growth rate $= 2.5\%$

ANSWER: **B**

## 8

$y = ab^{\frac{t}{p}}$ ← *Use this formula, **find** $p$*

$1160 = 3600\left(\frac{1}{2}\right)^{\frac{24}{p}}$

We were given amount after 1 day (answers are all in hours); **FIND** $p$; end amount; Initial amount; Half-life problem

$\frac{1160}{3600} = \left(\frac{1}{2}\right)^{\frac{24}{p}}$ *Isolate the power term*

$\log_{1/2}\left(\frac{1160}{3600}\right) = \frac{24}{p}$ *Convert to log form (or, if more to your liking, "log both sides")*

$p = \frac{24}{\log_{1/2}\left(\frac{1160}{3600}\right)}$ ➡ $p \approx 14.7$ *(**hours**, since $t$ was in hrs.)*

ANSWER: **B**

## 9

$y = ab^{\frac{t}{p}}$ ← *Use this formula, where $p = 1$*

$1127.84 = 1000(b)^2$ ← After two years

end amount; Initial amount; **FIND** $b$

$\frac{1127.84}{1000} = b^2$ *Isolate the power term*

$b^2 = 1.12784$

$b = \sqrt{1.12784}$ *Square root both sides*

$b \approx 1.062$ *Use this in equation*

*Now, use $b = 1.062$ to solve for "$t$" when $y = 2000$.*

$y = 1127.84(1.062)^t$

$2000 = 1127.84(1.062)^t$

$\frac{2000}{1127.84} = (1.062)^t$

*Convert to log form to solve for $t$:*

$t = \log_{1.062}\left(\frac{2000}{1127.84}\right)$

$t \approx 9.5$ ANSWER: **D**

## NR#3

Convert each to exp. form:

$a^{2b-1} = 8$  $2^b = a$

Substitute "$2^b$" for $a$ in first equation

$(2^b)^{2b-1} = 8$

Re-write using base 2:

$2^{2b^2-b} = 2^3$

Set exponents equal and solve:

$2b^2 - b = 3$ ➡ $2b^2 - b - 3 = 0$

$(2b-3)(b+1) = 0$  $b = 3/2$ or ~~$-1$~~ *extraneous*

ANSWER: **1.5**

## 10.

Get the log terms on the same side:

$\log_5 a - 2\log_5 c = b$

Apply log laws on left side:

$\log_5 a - \log_5 c^2 = b$

$\log_5 \frac{a}{c^2} = b$

Convert to exp. form to isolate "$a$":

$5^b = \frac{a}{c^2}$ ➡ $a = 5^b c^2$  ANSWER: **B**

## NR#4

Convert to exp. form to solve:

$(x+1)^2 = 2x + 10$

Expand left side, move right side terms over

$x^2 + 2x + 1 - 2x - 10 = 0$

Simplify and solve resulting quadratic:

$x^2 - 9 = 0$ ➡ $x^2 = 9$

$x = 3$ or ~~$-3$~~ ***Extraneous** – subbing into the original equation would lead to logging a negative (illegal)*

ANSWER: **13**

## 11.

Convert each to exp. form:
(first isolate power term on 2nd one)

$8^{2/3} = m$  $2^n = 5/3$ ➡ $n = \log_2\left(\frac{5}{3}\right)$

$m = 4$

Now, substitute into: $\log_m n^m$

$= 4\log_4\left(\log_2\left(\frac{5}{3}\right)\right)$ *Put into your calc*

```
4log₄(log₂(5/3))
        -0.8806616548
```

$\approx -0.88$ ANSWER: **C**

## 12.

Split up $360$ into multiples of **5** and **2** *(since we're given those logs)*

$= \log_3(5 * 8 * 9)$

Split up using log laws:

$= \log_3 5 + \log_3 8 + \log_3 9$

$= \log_3 5 + \log_3 2^3 + 2$  $3^{\square} = 2$ ?

$= \log_3 5 + 3\log_3 2 + 2$

$= \boldsymbol{a + 3b + 2}$ ANSWER: **D**

## NR#5

Convert to exp. form (using base 2, as indicated) to solve:

$\log_2(5^m) = 3m - 1$

Use log laws on left side:

$m\log_2 5 = 3m - 1$

Re-arrange get "$m$" terms on same side:

$1 = 3m - m\log_2 5$  Factor out "$m$" to isolate

$1 = m(3 - \log_2 5)$

$\frac{1}{(3 - \log_2 5)} = m$ ANSWER: **35**

## 13.

**Step 1** #'s in front become exponents:

$\log_2(4m)^{\frac{1}{2}} = \log_2 n + \log_2 p^2$

**Step 2** Combine LS to single log:

$\log_2\sqrt{4m} = \log_2(np^2)$

**Step 3** Since same base – drop logs:

$\sqrt{4m} = np^2$

➡ $2\sqrt{m} = np^2$ ➡ $\sqrt{m} = \frac{np^2}{2}$

**Step 4** Square both sides to isolate $m$

$(\sqrt{m})^2 = \left(\frac{np^2}{2}\right)^2$ ➡ $m = \frac{n^2p^4}{4}$

ANSWER: **A**

**NR #6** Log laws / combine to single log:

$$\log_5\left(\frac{x}{x-1}\right) = 3$$

Convert to exp. form:

$$5^3 = \frac{x}{x-1}$$

Cross-multiply to solve:

$$125(x-1) = x$$

$$125x - 125 = x$$

$$125x - x - 125 = 0$$

$$\mathbf{124}x - 125 = 0$$

ANSWER: **124**

**14.** Log laws / #'s *in front* become exponents:

$$= \log(4A)^2 - \log 2^3 + \log\left(A^{1/2}\right)^6$$

*(Also distribute – sign through brackets)*

$$= \log\left(4^2A^2\right) - \log 8 + \log A^3$$

Log laws / combine to single log:

$\log$ ——— *Pos (+) logs go up top* / *Neg ( − ) logs down here*

$$= \log\left(\frac{16A^2 * A^3}{8}\right)$$

Simplify inside brackets:

$$= \log\left(2A^5\right)$$

ANSWER: **B**

**NR #7** Express Richter values as exponents of 10:

$$\frac{10^{higher\ Richter}}{10^{lower}} = \text{"times as intense"}$$

$$\frac{10^{7.3}}{10^{x}} = 4$$

*Van. Island Richter is higher / goes on top*

*If Q. Charlotte had 1/4th intensity, then Van. Island was 4 times as intense*

*Unknown / Richter for Q.Charlotte earthquake was smaller (goes on bottom)*

$$10^{7.3-x} = 4$$

Convert to log form to solve:

$$\log_{10} 4 = 7.3 - x \quad \Rightarrow \quad x = 7.3 - \log 10$$

$$\Rightarrow \mathbf{x \approx 6.7}$$

ANSWER: **6.7**

**15.** Convert first log to exp. form:

$m^5 = n$ *We can now substitute into second log*

$$\log_m\left(n^{\frac{3}{4}}m^2\right)$$

Becomes....

$$\log_m\left(\left(m^5\right)^{\frac{3}{4}}m^2\right)$$

Now simplify:

*Multiply exponents:* $5 * \frac{3}{4}$

$$= \log_m\left(m^{\frac{15}{4}}m^2\right)$$

*Add exponents:* $\frac{15}{4} + 2 = \frac{15}{4} + \frac{8}{4}$

$$= \log_m\left(m^{\frac{23}{4}}\right)$$

$$= \mathbf{\frac{23}{4}}$$

ANSWER: **D**

**16.** Combine L.S. to single log:

$$\log_3[(x-3)(x-2)] = 2$$

Then convert to exp. form:

$$3^2 = (x-3)(x-2)$$

Solve resulting quadratic equation.

*But watch for extraneous solutions!*

$$9 = x^2 - 5x + 6$$

$$0 = x^2 - 5x - \mathbf{3}$$

ANSWER: **A**

## Written #1

*First bullet*

The horiz. asymptote represents the vert. shift, "$k$"

So.... $\mathbf{k = 2}$

Sub into equation and solve for "$a$"

$$y = a(3)^x + 2$$

Use any given point to solve for "$a$"

$$\mathbf{7} = a(3)^{\mathbf{0}} + 2 \Rightarrow 7 = a(1) + 2 \Rightarrow \mathbf{a = 5}$$

So equation is:

$$\mathbf{y = 5(3)^x + 2}$$

*Second*

For inverse, switch $x$ and $y$

$f(x)$: $y = 3(2)^x - 1$

$g(x)$: $\mathbf{x} = 3(2)^{\mathbf{y}} - 1$

Then solve for $y$:

Isolate power term, $2^y$, to convert to log form

$$x + 1 = 3(2)^y \Rightarrow \frac{x+1}{3} = 2^y \Rightarrow \log_2\left(\frac{x+1}{3}\right) = y \Rightarrow$$

$$\mathbf{y = \log_2\left(\frac{x+1}{3}\right)} \quad \text{or} \quad \mathbf{y = \log_2\left[\frac{1}{3}(x+1)\right]}$$

*Third bullet*

$g(x)$ has an $x$-int when $y = 0$

*Using* $x = 3(2)^y - 1$ *equation:*

$$x = 3(2)^{\mathbf{0}} - 1 \quad x\text{-intercept}$$

$$x = 3(1) - 1 \Rightarrow \mathbf{x = 2}$$

*Using* $y = \log_2\left(\frac{x+1}{3}\right)$ *equation:*

$$\mathbf{0} = \log_2\left(\frac{x+1}{3}\right)$$

$$\Rightarrow 2^0 = \frac{x+1}{3}$$

*Same result!* $3 = x + 1$

$g(x)$ has a $y$-int when $x = 0$

*Using* $y = \log_2\left(\frac{x+1}{3}\right)$ *equation:*

$$y = \log_2\left(\frac{\mathbf{0}+1}{3}\right) \Rightarrow \mathbf{y = \log_2\left(\frac{1}{3}\right)}$$

## Written #2

**First bullet**

Use $y = ab^{t/p}$ to solve for $p$:

$$0.959 = 1.220\left(\frac{1}{2}\right)^{\frac{10}{p}}$$

← Use values from table, after 10 years amount went from 1.220 to 0.959

*Isolate power term:*

$$\frac{0.959}{1.220} = \left(\frac{1}{2}\right)^{\frac{10}{p}}$$

*Then convert to exp. form to solve:*

$$\frac{10}{p} = \log_{1/2}\left(\frac{0.959}{1.220}\right) \Rightarrow p = \frac{10}{\log_{0.5}\left(\frac{0.959}{1.220}\right)} \Rightarrow p \approx 28.8 \text{ years}$$

Half-life

Equation that models amount of Strontium-90:

$$y = 1.220\left(\frac{1}{2}\right)^{\frac{t}{28.8}}$$

*Alternatively you could "log both sides"*

$$\log\frac{0.959}{1.220} = \log\left(\frac{1}{2}\right)^{\frac{10}{p}} \Rightarrow \log\left(\frac{0.959}{1.220}\right) = \frac{10}{p}\log\left(\frac{1}{2}\right) \Rightarrow p = \frac{10\log\left(\frac{1}{2}\right)}{\log\left(\frac{0.959}{1.220}\right)}$$

*Same result, $p \approx 28.8$*

**Second**

Initial percentage is 100, so use $A_0 = 100$

$$10 = 100(0.9172)^t$$

*Isolate power term:*

$$\frac{10}{100} = (0.9172)^t$$

$$0.1 = (0.9172)^t$$

*Then convert to exp. form to solve:*

$$t = \log_{0.9172}(0.1) \Rightarrow t \approx 26.6 \text{ days}$$

*Alternatively you could "log both sides"*

$$\log 0.1 = \log 0.9172^t$$

$$\log 0.1 = t\log 0.9172$$

$$\frac{\log 0.1}{\log 0.9172} = t$$

**Third bullet**

"bonus"!

The half-life is the period of time ("$p$" in the equation) necessary for the initial amount to drop to half.

$$0.5 = 1(0.9172)^t$$

← *Make the initial amount $1$, and the end amount $0.5$*

*Then convert to exp. form to solve:*

$$t = \log_{0.9172}(0.5)$$

$$t \approx 8.02 \text{ days} \Rightarrow A = 100\left(\frac{1}{2}\right)^{\frac{t}{8.02}}$$

*Alternate equation using half-life of 8.02 days*

**And with that – you're done!**


(Or try another practice exam)